

以 Bohr 模型導出能量關係式

Centripetal force = Coulombic attraction

$$\frac{mv^2}{r} = \frac{ze^2}{r^2}$$

$$\rightarrow mv^2 = \frac{ze^2}{r} \quad (1)$$

Quantization of angular momentum:

$$mvr = n\hbar \quad (2)$$

(1)/(2)

$$\rightarrow v = \frac{ze^2}{n\hbar}$$

代入式 (2)

$$mr \frac{ze^2}{n\hbar} = n\hbar$$

$$\rightarrow \frac{1}{r} = \frac{mze^2}{n^2\hbar^2} \quad (3)$$

$$\text{Total energy } E = \frac{1}{2}mv^2 - \frac{ze^2}{r}$$

$$\text{將式(1)代入上式得} \quad E = -\frac{ze^2}{2r}$$

$$\rightarrow E = -\frac{mz^2e^4}{2n^2\hbar^2} = -\frac{2\pi^2mz^2e^4}{n^2h^2} = \frac{E_{n=1}}{n^2}$$

$$E = -2.178 \times 10^{-18} \frac{z^2}{n^2} \text{ J}$$

n: integer

$$n^2 \propto r$$

For hydrogen atom: $E = -2.178 \times 10^{-18} \left(\frac{1}{n^2} \right) \text{J}$

From $n = 6 \rightarrow n = 1$

$$E_{n=6} = -2.178 \times 10^{-18} \left(\frac{1}{6^2} \right) \text{J}$$

$$E_{n=1} = -2.178 \times 10^{-18} \left(\frac{1}{1^2} \right) \text{J}$$

$$\begin{aligned} \Delta E = E_{n=1} - E_{n=6} &= -2.178 \times 10^{-18} \left(\frac{1}{1^2} - \frac{1}{6^2} \right) \text{J} \\ &= -2.118 \times 10^{-18} \text{J} \end{aligned}$$

$$\Delta E = h \frac{c}{\lambda}$$

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.626 \times 10^{-34})(2.9979 \times 10^8)}{2.118 \times 10^{-18}} = 9.379 \times 10^{-8} \text{ m}$$

From $n = 1 \rightarrow n = 2$

$$\begin{aligned} \Delta E = E_{n=2} - E_{n=1} &= -2.178 \times 10^{-18} \left(\frac{1}{2^2} - \frac{1}{1^2} \right) \text{J} \\ &= 1.634 \times 10^{-18} \text{J} \end{aligned}$$

$$\lambda = 1.216 \times 10^{-7} \text{ m} = 121.6 \times 10^{-9} \text{ m} = 121.6 \text{ nm}$$

For hydrogen: $n = 5 \rightarrow n = 2$ blue

$n = 4 \rightarrow n = 2$ green

$n = 3 \rightarrow n = 2$ red

Overall

$$\Delta E = E_{\text{final}} - E_{\text{initial}} = -2.178 \times 10^{-18} \left(\frac{1}{n_{\text{final}}^2} - \frac{1}{n_{\text{initial}}^2} \right) \text{J}$$

Q: Minimum E to remove e^- in H atom from its ground state?

From $n = 1 \rightarrow n = \infty$

$$\begin{aligned}\Delta E = E_{n=\infty} - E_{n=1} &= -2.178 \times 10^{-18} \left(\frac{1}{\infty^2} - \frac{1}{1^2} \right) \text{ J} \\ &= 2.178 \times 10^{-18} \text{ J}\end{aligned}$$