1. The paradox of the pole vaulter

A pole vaulter runs with a constant velocity v with respect to a bar which has two doors that are 10 m apart according to observer B. The pole length measured by observer A is also 10 m long. The front end of the pole enters the first door at event E_0. The back end of the pole clears the first door at event E_2. Event E_1 marks the meeting of the front of the pole with the second door. Since event E_2 occurs before E_1 to the barn observer (observed in B-frame), the two doors can be shut so as to contain the pole vaulter entirely within the barn. The view of the runner is that when the pole breaks the second door at event E_1, the back end of the pole is still outside the first door. At this instant (in A-frame), the door is represented by the event E_3 and the back end of the pole by E_4.

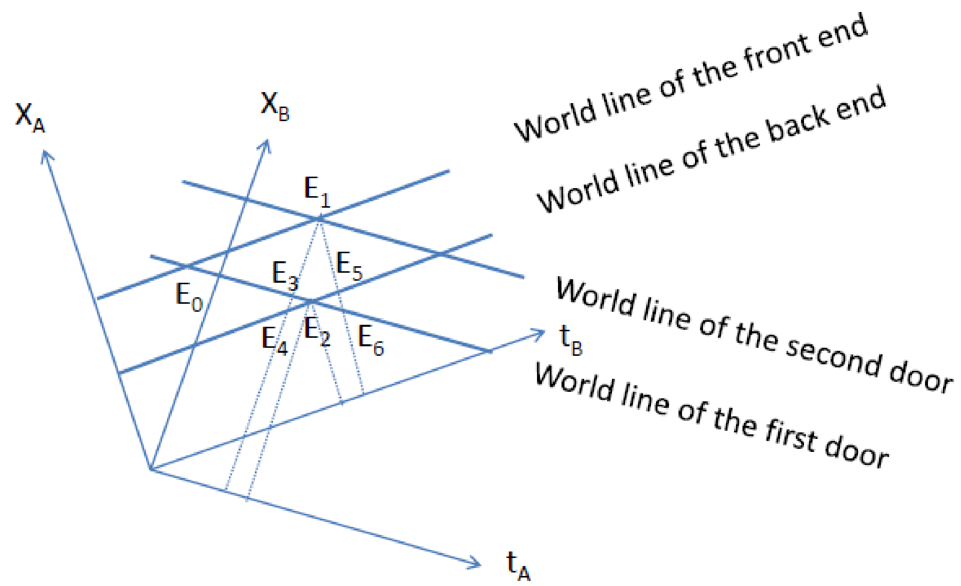

Fig. 1: The Lorentz diagram of the pole vaulter and the barn.

The "paradox" is perhaps our best illustration of the importance of f stressing the event in relativistic problems. The events E_0, E_1 and E_2 really happen. All observers agree that the ends of the pole pass through the barn doors. The problem posed by the paradox is to resolve the time order in which these events occur. Rather than attempt to discuss the paradox by comparing the relative lengths of the pole and barn at given instants, we first identify the events in the space-time graph. Once we have established the time and position of these events in one frame of reference, we can make the Lorentz transformation to any other frame.

2. Visual appearance of the a moving rod

The relativistic contraction of length is unlikely to be seen directly with the unaided eye because the speed of the object relative to the observer must be near that of the speed of light. Even though such a relative speed be reached, the object would move by so fast as to be virtually invisible.

There is a more subtle effect, however, which prevents us from detecting the relative contraction by looking at the moving object, except in very special circumstances. We recall from the discussion of the Doppler effect that the apparent duration of a phenomenon is influenced by the changing the transit time of the light by which the observer sees the phenomenon. A similar effect enters in the visual observation of the length of a moving object.

Imagine that the object is transparent and approaches the observer at a high speed. The image of the object in the observer's eye is formed from light that arrives simultaneously from the far and near sides of the object. The light arriving from the far side therefore leaves its sources earlier than the light from the near side. Because the object is approaching the observer, the light from the far end crosses a distance greater than the length of the object before it joins the light from the near end and proceeds on to the observer. If the observer can sense depth, he thinks the objects is longer than it really is.

For a geometric analysis of the problem, consider Fig. 2. The A-frame is the moving rod with a length of L_A in the A-frame. The B-frame is the rest frame of the observer. At event E_1, light leaves the far end of the rod, which is assumed to have a refractive index of unity, and joins light emitted from the near end at event E_2. The two light arrive at the observer at event E_3. Since the line segment E1E2 is part of the world line of a light signal, the visual length of the approaching rod in the B-frame is

$$L'_B = c\,\Delta t_B\,.$$

During the time interval Δt_B, the rod has moved forward a distance Δx_B in the B-frame. That is $\Delta x_B = c\,\Delta t_B \sin\alpha = v\Delta t_B = L'_B\,(v/c)$. The visual length is given by

$$L'_B = L_B + \Delta x_B = L_B + L'_B\,(v/c).$$

$$L'_B = L_B/(1\text{-}v/c)$$

Under Galilean transformation, $L_B = L_A$, so that the classical visual length of the rod is

$$L'_B = L_A/(1\text{-}v/c)$$

Under Lorentz transformation, $L_B = L_A \cos\alpha = L_A\sqrt{1 - v^2/c^2}$, so that the relativistic visual length of the rod is given by

$$L'_B = \quad L_A \frac{\sqrt{1-v^2/c^2}}{1-v/c} = L_A \frac{\sqrt{1+v/c}}{\sqrt{1-v/c}} \qquad \text{(approaching).}$$

It's not difficult that the corresponding equation for the receding rod is

$$L'_B = \quad L_A \frac{\sqrt{1-v^2/c^2}}{1+v/c} = L_A \frac{\sqrt{1-v/c}}{\sqrt{1+v/c}} \qquad \text{(receding).}$$

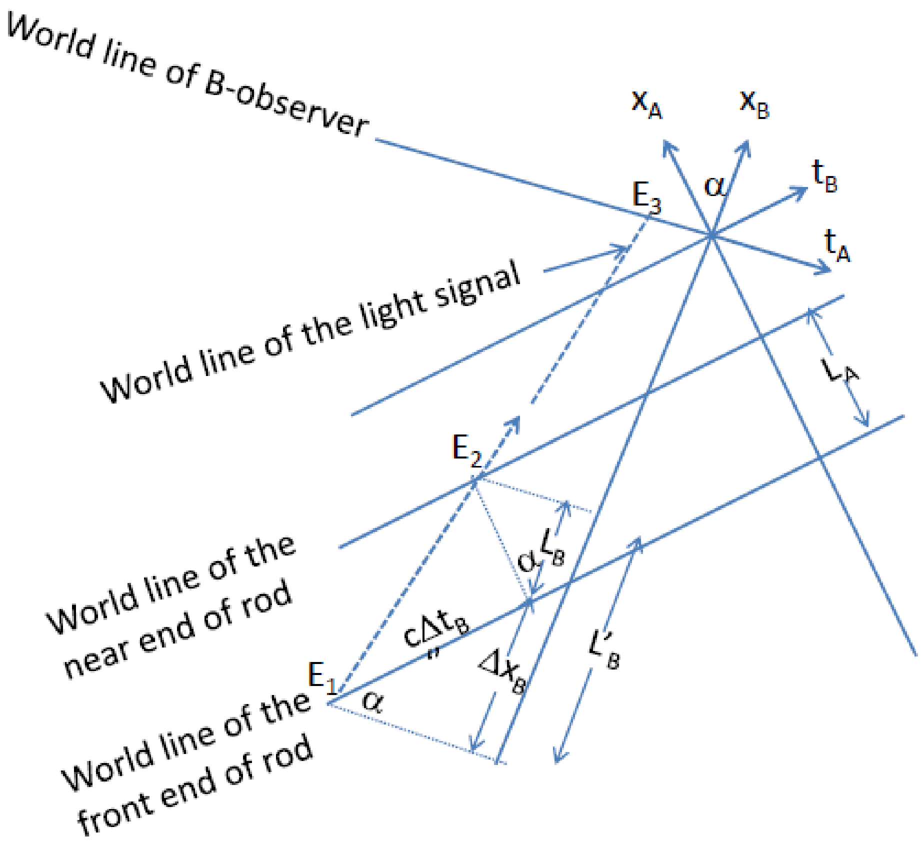

Fig. 2: A transparent rod approaching an observer is seen to be longer than its rest length.