

- **Example 1 (Simultaneity)** The relativity of simultaneity is closely connected to the invariance of the speed of light. Imagine two trains A and B moving relatively at half of the speed of light. A flash bulb is triggered in the exact center of one of the A-train cars. Since the speed of light is the same in all directions to the A-observer, the light strikes the ends of the car, which are 10 m away from the center., at the same instant in the A-frame. However, to the B-observer, the A-car moves forward so that its front wall moves away from the advancing light signal and its back wall moves toward the light. Since the speed of light to B is unaffected by the relative motion of A, the light strikes the back wall before it strikes the front wall.

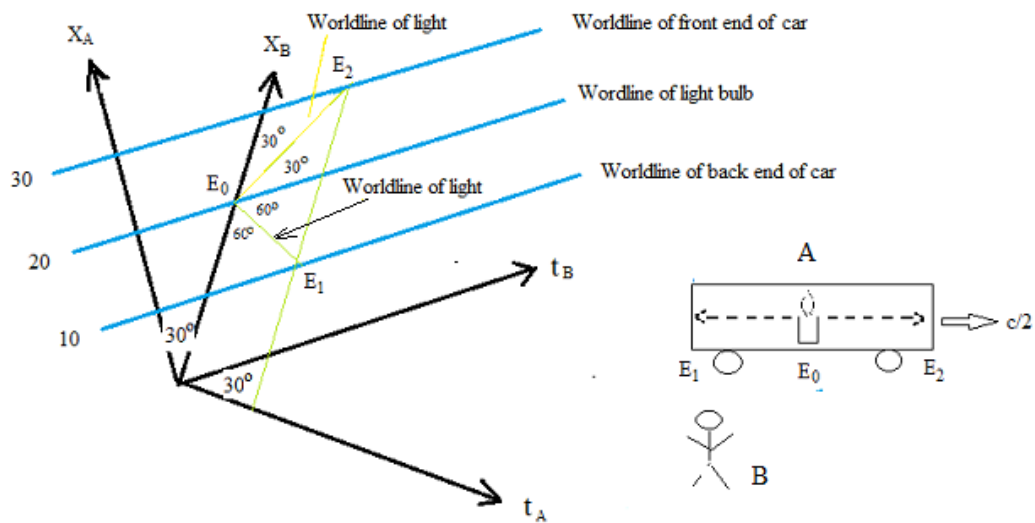


Fig. 1: Two light beams strike the car walls simultaneously in the A-frame but not in the B-frame.

Figure 1 is the Loedel diagram for this problem. The car is at rest in the A-frame. The spatial origin of the A-system is located 10 m behind the back end of this car, so the world line of the light bulb is perpendicular to the X_A -axis at $X_A = 20$ m. Event E_0 marks the flash bulb, which occurs at $t_A = 0$. The backward-moving beam is represented by the world line of E_0E_1 and the forward-moving beam by E_0E_2 . Events E_1 and E_2 represent the light striking the back and front ends of the car, respectively. The world lines of light, it is recalled, are always oriented so that their projections on to space and time axes of a given frame are equal. The line E_0E_2 has this property because it is drawn parallel to the bisector of the 60° angle between X_B - and t_B -axes. This bisector also bisects the 120° angle between the X_A - and t_A -axes. The line E_0E_1 is also a light world line because it is drawn at right angles of the angle bisector. Therefore, events E_1 and E_2 lie on a perpendicular to the t_A -axis and are simultaneous in the A-frame. However, E_1 and E_2 are not simultaneous in the B-frame.

The A-time needed for the light to transverse half of the car length is clearly

$$\Delta t_A = 10 \text{ m} / (3 \times 10^8 \text{ m/sec}) = 3.33 \times 10^{-8} \text{ sec.}$$

The B-time in which the light travels from the bulb to the advancing rear wall if the car is $\Delta t_{1B} = 10 \text{ m} \times \tan(30^\circ) / (3 \times 10^8 \text{ m/sec}) = 1.92 \times 10^{-8} \text{ sec.}$

At longer B-time elapses while the light travels from the bulb to the retreating front wall of the car:

$$\Delta t_{2B} = 10 \text{ m} \times \cot(30^\circ) / (3 \times 10^8 \text{ m/sec}) = 5.77 \times 10^{-8} \text{ sec.}$$

The time separating events E_1 and E_2 in the B-frame is

$$\Delta t_{12B} = (5.77 - 1.92) \times 10^{-8} \text{ sec.}$$

- Example 2 (Time dilation)

For an illustration of time dilation, we let the A-clock be the moving clock. It's motion is relative to the B-frame, in which there is an array of B-clocks distributed at rest along the X_B -axis. As the A-clock passes each of the B-clock, a record is made of the readings of the A-clock and the corresponding B-clock. This record therefore contains the readings of a single A-clock placed in one-to-one correspondence with an appropriate readings of an array of B-clocks. The A-clock is read many times while each B-clock is read only once.

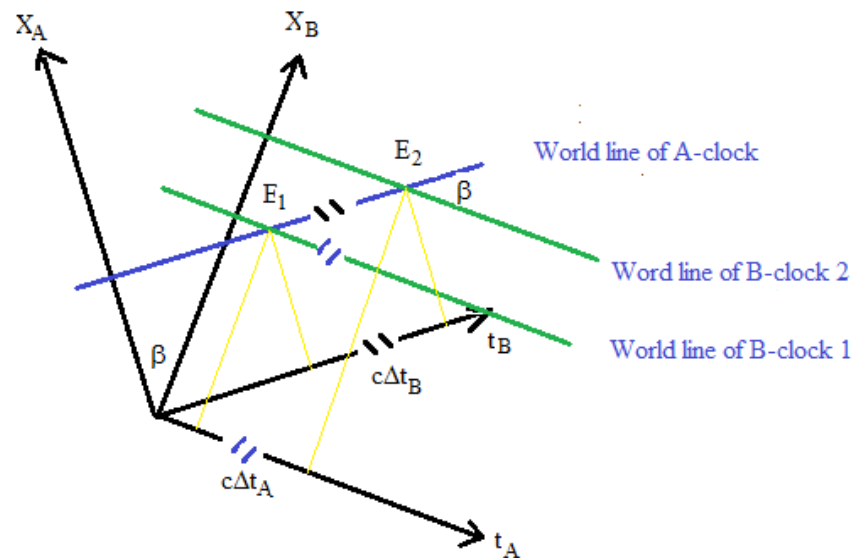


Fig. 2: Time dilation arises from comparison of the readings of a moving clock with readings at rest.

In Fig. 2, the world line of the A-clock is perpendicular to the X_A -axis. It intersects the world lines of two B-clocks at events E_1 and E_2 . The time elapsed between these events, as measured by the A-clock is Δt_A . The time elapsed between E_1 and E_2 , as measured in B-frame cannot be computed from the readings of a single B-

Clock. Because E_1 and E_2 occurs at different places in the B-frame. The time interval Δt_B is obtained by subtracting the reading of the B-clock at event E_1 from the reading of the B-clock at event E_2 . The diagram shows that

$$c\Delta t_A = c\Delta t_B \cos\beta, \Delta t_B = \frac{\Delta t_A}{\sqrt{(1-v^2/c^2)}} \quad (1)$$

The time interval Δt_A is shorter than Δt_B . Figure 3 represents a composite of several consecutive photographs taken by a camera carried with the A-clock. The camera periodically takes a picture of the A-clock reading and reading of B-clock going by at that instant.

t_B (sec)	0	15.0	30.0	45.0
B-clock	↑	→	↓	←
A-clock	↑	↗	→	↘
t_A (sec)	0	7.5	15.0	22.5
x_A ($\times 10^8$ m)	100	100	100	100

Fig. 3: The reading of a single A-clock is compared with the readings of the B-clocks it momentarily passes.

Let us change the “chosen frame” of reference from the B-frame to the A-frame and again make a comparison of clock rates. There is now only a single B-clock and an array of A-clocks distributed along the x_A -axis. As the B-clock moves in the negative direction along the x_A -axis, its readings are matched to those of the A-clock it passes. An analysis identical to the one above leads to the conclusion that B-clock runs more slowly than the A-clocks.

Time dilation seems to lead a contradictory results. How can A-clocks run more slowly than B-clocks while B-clocks run more slowly than A-clocks? The answer lies in the choice of reference frame. Let’s return to the first example, in which the comparison of clock rates is made relative to the B-frame. It is not correct to say

simply that the A-clock goes slower than the B-clock, because two B-clocks are involved in arriving at Eq. 1. To make the statement correct, we must state the frame relative to which simultaneity is defined. From the point of view of an A-observer, the A-clock passes by B-clocks which are not synchronized relative to the A-frame, and which are therefore increasingly out of phase relative clocks in the A-frame. The A- and B-observers both agree that the record of A-clock readings versus B-clock readings is correct. The B-observer, with his interpretation of simultaneity claims that the record proves that the A-clock runs slower than his clocks. The A-observer thinks that the record proves that the B-clocks are not synchronized. Another set of data is needed to justify the A-observer's claim that a given B-clock runs slower than an A-clock. These data are the record made by a single B-clock and an array of A-clocks. The observer interprets these data to mean that the A-clocks are not synchronized.